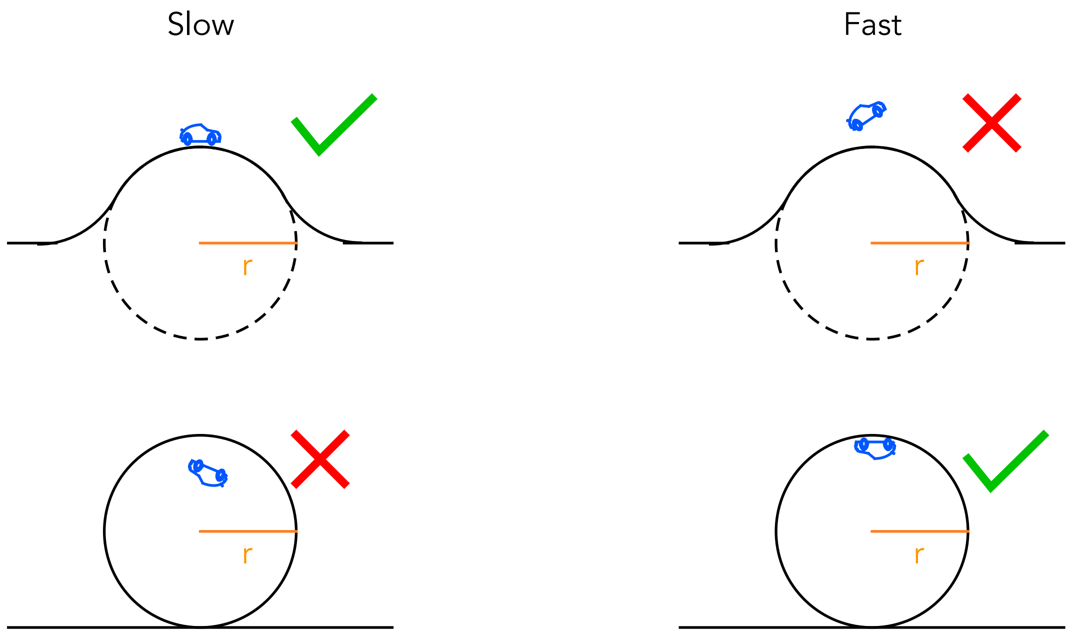
Vertical Circles

There are two ways one can travel in a vertical circle. First, you could always be right-side-up, like driving a car over a hill or riding a Ferris Wheel. Second, you could be upside-down, like a loop-the-loop roller coaster or twirling a bucket of water. In the first case, you could go as slow as you like and be fine, but if you go too fast you will become a projectile. In the second case, you have to be going pretty fast in order to safely make the loop, and if you go too slow you will fall down (or get wet with a bucket of water.) These situations are shown in the diagram below:



An obvious question in the first case is "what is the *fastest* you can go over the hill and just barely stay on the hill?" In the second case, the obvious question is "what is the *slowest* you can go on the loop and just barely make the loop?" It turns out, that the ideas behind both of those answers are the same – and so the answers are the same! Let's go through this.

1. Let's focus on the forces for the two "happy" results. In both cases, the same two forces are acting on the car. What are they?

2. In both cases, the net force on the car is in the same direction. What is that direction?

3. For the car going slowly over the hill, draw the free-body diagram.

4. For the car going quickly through the loop, draw the free-body diagram.

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5. In both cases, one of the forces acting on the car *doesn't* depend on the speed of the car. Which is it and does it point in the same direction as the net force or the opposite direction?

6. If the car is at rest on the top of the hill, what is the net force on the car and what is the normal force (compared to the weight of the car?)

7. For the car going over the hill, does the normal force *help* the car go in the circle or does it *fight* the car going in a circle? (Think about the direction of the normal force compared to the centripetal force.)

8. So as the car goes a little bit faster on the hill, what happens to the centripetal force and what happens to the normal force?

9. When the car is going the fastest it can on the hill, the centripetal force is the biggest it can be and the normal force is the smallest it can be. What is the smallest the normal force can be, and so what is the biggest the centripetal force can be?

10. Now, for the car going through the loop, does the normal force *help* the car go in the circle or does it *fight* the car going in a circle? (Think about the direction of the normal force compared to the centripetal force.)

11. Since the normal force always helps to push the car down and always helps to keep it in a circle, the car can actually go as fast as it likes around the loop. The faster it goes, what happens to the normal force?

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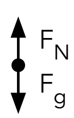
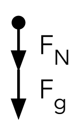
12. When the car is going the slowest it can on the loop, the centripetal force is the smallest it can be and the normal force is the smallest it can be. What is the smallest the normal force can be, and so what is the smallest the centripetal force can be?

13. Notice that in both cases (the fastest over the hill and the slowest through the loop) the centripetal force is just the force of gravity! Set the force of gravity equal to the centripetal force find how the cutoff speeds depend on the radius of the circle.

Answers:

1) *force of gravity & normal force*
2) *down, the center of the circle is below the car in both cases*
3) 

4) 

5) *the force of gravity & it points down, so the same direction as the net force*
6) *net force is 0 & the normal force would be the same as the force of gravity (to cancel it out.)*
7) *it fights it because the normal force is pointed in the opposite direction.*
8) *the centripetal force gets bigger and the normal force gets smaller*
9) *the smallest normal force would be 0 & so the biggest centripetal force would be just the force of gravity*
10) *it helps because the normal force is pushing down*
11) *the normal force would get bigger the faster the car goes through the loop*
12) *the smallest normal force would be 0 & so the smallest centripetal force would be just the force of gravity*
13) *so* $mg = mv^2/r$ *turns into* $v^2 = rg$